Further, Eqs. (29)-(31) give

$$(\rho_1^0\alpha_1+\rho_2^0\alpha_2)\left(\frac{m_1}{m_1+1}\right)\left(\frac{\rho F_l}{k_1^*}\right)^{\frac{1}{m_1}}\delta_2^{\frac{2m_1+1}{m_1}}+\frac{d^2(\alpha_{20}-\alpha_2)\rho_2^{0\,2}\alpha_2 F_l}{Ak}\delta_2=\frac{q_1+q_2}{2\pi r}, \qquad (33)$$

$$(\rho_1^0\alpha_1)\left(\frac{m_1}{2m_1+1}\right)\left(\frac{\rho F_l}{k_1^*}\right)^{\frac{1}{m_1}}\delta_2^{\frac{2m_1+1}{m_1}}=\frac{q_1}{2\pi r}, \qquad (34)$$

from which $\delta_2(l)$ and $\alpha_2(l)$ are determined.

Note that if pure liquid flows over the rotor surface, Eq. (32) becomes the solution obtained in [4].

It is also possible to use the Rakhmatulin interpenetration model, together with experimental data, to calculate the flow of materials in other mixers, centrifuges, centrifugal diffuser-atomizers, etc.

NOTATION

V_j, ρ_j, α_j, velocity, mean density, and concentration (by volume) of the j-th phase; ρ_j^0, true density of the j-th phase; **T**, liquid stress tensor; F_j, mass force acting on the j-th phase; τ_1^{ki}, and e^{ki}, stress and strain-rate tensors; f_{12}, phase-interaction force; P, pressure; R, radius of conical rotor channel; x^i, orthogonal coordinates; ρ, V, density and velocity of mixture; η, effective liquid viscosity; d, characteristic dimension of solid particles; ω, angular velocity of rotor; k, k*, n, m, m_1, k_1^*, power-law parameters for liquid and mixture; α, semivertex angle of conical channel; W, collective rate of settling of solid particles; Φ_S, factor determined by the shape of the solid particles; q_j, mass flow rate of j-th phase; $r = R-\delta\cos\alpha$, distance from axis of rotor rotation to an arbitrary point; ρ_{20}, bulk density of solid phase.

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CHARACTERISTICS OF FLOW BETWEEN A ROTATING AND A STATIC DISK IN THE PRESENCE OF RADIAL FLOW

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An improved method is proposed for the calculation of the flow in the gap between a rotating and a static disk in the presence of radial flow. The algorithm of the solution is realized on a Nairi-2 computer.

To solve a number of problems associated with the hydraulic circulation section of a multistage turbine with disk rotors and, in particular, to calculate the axial forces and temperature state of the rotors of a steam turbine, it is necessary to know the radial distribution of the pressure of the medium in the gap between a rotating disk and the corresponding static element (diaphragm, casing). An approximate solution of this problem was obtained in [1] and subsequently refined in [2-4]. In [5], there was further development of the method of calculating the pressure distribution along the disk radius in the presence of radial flow, but the

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velocity profile used for the flow in the gap led, under certain conditions, to the appearance of negative values of the flow swirl, which is inconsistent with the physical interpretation of the problem. Consequently, the agreement between the results obtained according to [5] and experimental data is limited [6].

In [6], on the basis of a theoretical analysis and experimental data, a calculation based on similarity theory was proposed. However, this approach requires the availability of data obtained in model experiments.

In a number of works — in particular, [7] — it has been shown by experiment and calculation that in a broad range of radial flow rates the profile of the radial component of the flow velocity is analogous to the velocity profile in a radial diffuser. In the present paper, this analogy is applied to the calculation of the radial pressure distribution in the gap between a rotating and a static disk.

It is assumed that the medium in the gap between the disks is incompressible and that the flow is axisymmetric. In the case when the gap between the disks is narrow, i. e., the width s is considerably less than the length r_2-r_1, the time-averaged turbulent flow in the gap can be described by the equations

$$\frac{1}{r}\cdot\frac{d}{dr}r\int_0^s c_r^2\,dz-\frac{1}{r}\int_0^s c_\varphi^2 dz=-\frac{s}{\rho}\cdot\frac{dp}{dr}+\frac{\tau_{zr}}{\rho}\Big|_0^s; \tag{1}$$

$$\frac{1}{r^2}\cdot\frac{d}{dr}r^2\int_0^s c_r c_\varphi dz=\frac{\tau_{z\varphi}}{\rho}\Big|_0^s; \tag{2}$$

$$2\pi r\int_0^s c_r dz=q. \tag{3}$$

Following [5], it is assumed that if the relative gap $\bar{s}=s/r_2\leq 0.1$, the flow between the disks is viscous, i. e., there is no potential nucleus of the flow, and that the boundary layers at the rotating and static disks are of thickness $\delta=\delta'=s/2$. In accordance with the results of [7], the profiles of the azimuthal and radial velocity components are written in the following form: close to the rotating disk,

$$\left.\begin{aligned} c_\varphi&=\omega r\left[1-(1-y)\left(\frac{z}{\delta}\right)^{\frac{1}{n_1}}\right]\\ c_r&=c_{r\frac{s}{2}}\left(\frac{z}{\delta}\right)^{\frac{1}{n_1}}\end{aligned}\right\}, \tag{4}$$

and close to the wall (static disk),

$$\left.\begin{aligned} c_\varphi&=\omega r y\left(\frac{s-z}{\delta}\right)^{\frac{1}{n_2}}\\ c_r&=c_{r\frac{s}{2}}\left(\frac{s-z}{\delta}\right)^{\frac{1}{n_2}}\end{aligned}\right\}. \tag{5}$$

The stress–tensor components and the components of the flow velocity for the tube are related by a power law with index 1/n [8]:

close to the disk,

$$\frac{w}{c^*}=A_1\left(\frac{c^* r}{\nu}\right)^{\frac{1}{n_1}}, \tag{4a}$$

and close to the wall,

$$\frac{u}{c^*}=A_2\left[\frac{c^*(s-r)}{\nu}\right]^{\frac{1}{n_2}}. \tag{5a}$$

Let α denote the ratio of the radial and azimuthal components of the relative velocity and the corresponding stress–tensor components in the boundary layer close to the disk:

$$\alpha=\frac{c_{r\frac{s}{2}}}{\omega r(1-y)}=\frac{\tau_{zr}}{\tau_{z\varphi}}.$$

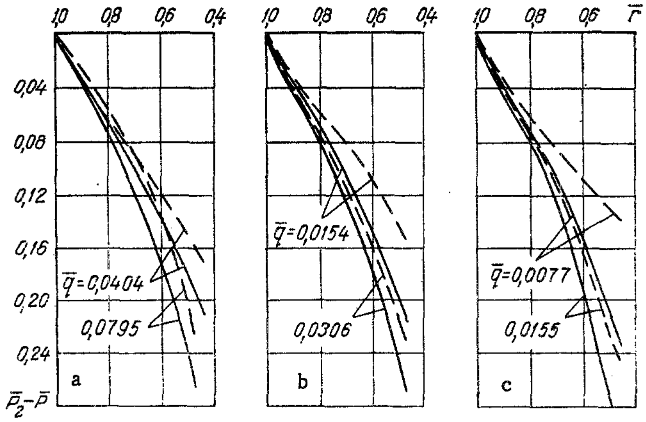

Fig. 1. Change in pressure in gap between rotating and static disks for flow from periphery to center ($Re_u = 2 \cdot 10^6$). The continuous curves correspond to calculation, and the dashed curves, to experiment [6]: a) $\bar{s}$ = 0.0114; b) 0.03; c) 0.06.

Then

$$\tau_{zr} = \frac{\alpha}{\sqrt{1+\alpha^2}}\tau_0; \quad \tau_{z\varphi} = -\frac{1}{\sqrt{1+\alpha^2}}\tau_0.$$

For the chosen velocity profile the relative velocity in the boundary layers may be expressed in terms of the flow velocity at the midpoint of the gap: close to the disk,

$$w = U\left(\frac{z}{\delta}\right)^{\frac{1}{n_1}}$$

and close to the wall,

$$w = U\left(\frac{s-z}{\delta}\right)^{\frac{1}{n_2}}$$

Taking this into account, together with the relation

$$U = \sqrt{c^2_{r\frac{s}{2}} + (\omega r - c_{\varphi\frac{s}{2}})^2} = \omega r(1-y)(1+\alpha^2)^{\frac{1}{2}},$$

Eqs. (4a) and (5a) give the result

$$\tau_0 = \rho A_1^{-\frac{2n_1}{1+n_1}}[\omega r(1-y)]^{\frac{2n_1}{1+n_1}}\left(\frac{\nu}{\delta}\right)^{\frac{2}{1+n_1}}(1+\alpha^2)^{\frac{n_1}{1+n_1}}.$$

Hence for the disk

$$\tau_{zr}|_{z=0} = \alpha\rho A_1^{-\frac{2n_1}{1+n_1}}[\omega r(1-y)]^{\frac{2n_1}{1+n_1}}(1+\alpha^2)^{\frac{n_1-1}{2(n_1+1)}}\left(\frac{\nu}{\delta}\right)^{\frac{2}{1+n_1}}; \tag{6}$$

$$\tau_{z\varphi}|_{z=0} = -\rho A_1^{-\frac{2n_1}{1+n_1}}[\omega r(1-y)]^{\frac{2n_1}{1+n_1}}(1+\alpha^2)^{\frac{n_1-1}{2(n_1+1)}}\left(\frac{\nu}{\delta}\right)^{\frac{2}{1+n_1}}. \tag{7}$$

Similarly, for the static disk

$$\beta = \frac{c_{r\frac{s}{2}}}{\omega r y};$$

$$\tau_{zr}|_{z=s} = \beta\rho A_2^{-\frac{2n_2}{1+n_2}}[\omega r y]^{\frac{2n_2}{1+n_2}}\left(\frac{\nu}{\delta}\right)^{\frac{2}{1+n_2}}(1+\beta^2)^{\frac{n_2-1}{2(n_2+1)}}; \tag{8}$$

$$\tau_{z\varphi}|_{z=s} = \rho A_2^{-\frac{2n_2}{1+n_2}}[\omega r y]^{\frac{2n_2}{1+n_2}}\left(\frac{\nu}{\delta}\right)^{\frac{2}{1+n_2}}(1+\beta^2)^{\frac{n_2-1}{2(n_2+1)}}. \tag{9}$$

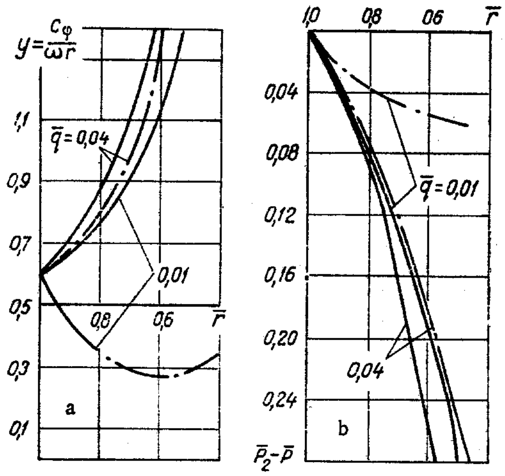

Fig. 2. Flow swirl and pressure variation between rotating and static disks for flow from periphery to center ($Re_u = 2 \cdot 10^6$; $\bar{s} = 0.0725$). The continuous curves correspond to calculation by the proposed algorithm and the dashed-dot curves, to calculation according to [5]: a) flow swirl; b) pressure drop.

Using a theorem on the integral calculation of the means, the left-hand side of Eq. (2) is transformed, assuming that the mean-integral value of the azimuthal velocity component is close to its value at the midpoint of the gap [3]:

$$\int_0^s c_r c_\varphi dz \approx c_{\varphi \frac{s}{2}} \int_0^s c_r dz = \omega y \frac{q}{2\pi}. \tag{10}$$

Thus Eq. (2) takes the form

$$\frac{\omega q}{2\pi} \cdot \frac{1}{r^2} \cdot \frac{d}{dr}(r^2 y) = \frac{1}{\rho} \tau_{z\varphi} \Big|_0^s. \tag{11}$$

Then Eqs. (3) and (5) give

$$c_{r \frac{s}{2}} = \frac{q}{\pi r s \left(\frac{n_1}{1+n_1} + \frac{n_2}{1+n_2} \right)}. \tag{12}$$

In addition,

$$\left. \begin{aligned} \alpha &= \frac{q}{\pi \omega r^2 s (1-y) \left(\frac{n_1}{1+n_1} + \frac{n_2}{1+n_2} \right)} \\ \beta &= \frac{q}{\pi \omega r^2 s y \left(\frac{n_1}{1+n_1} + \frac{n_2}{1+n_2} \right)} \end{aligned} \right\} \tag{12a}$$

Substituting Eqs. (4) and (5) into Eqs. (1) and (11), taking into account Eqs. (12) and (12a), and passing to dimensionless variables gives

$$\frac{dy}{d\bar{r}} = 2^{\frac{2}{1+n_1}} A_1^{-\frac{2n_1}{1+n_1}} (\bar{s}\mathrm{Re}_u)^{-\frac{2}{1+n_1}} \bar{r}^{-\frac{2n_1}{1+n_1}} (1-y) \frac{1}{\bar{s}\bar{q}} \left[(1-y)^2 + \left(\frac{2\bar{q}}{\bar{r}^2 \left(\frac{n_1}{1+n_1} + \frac{n_2}{1+n_2} \right)} \right)^2 \right]^{\frac{n_1-1}{2(n_1+1)}}$$

$$-2^{\frac{2}{1+n_2}}\frac{(\bar{s}\mathrm{Re}_u)^{-\frac{2}{1+n_2}}}{\bar{s}\bar{q}}A_2^{-\frac{2n_2}{1+n_2}}\bar{r}^{-\frac{2n_2}{1+n_2}}y\left[y^2+\left(\frac{2\bar{q}}{\bar{r}^2\left(\frac{n_1}{1+n_1}+\frac{n_2}{1+n_2}\right)}\right)^2\right]^{\frac{n_2-1}{2(n_2+1)}}-\frac{2y}{\bar{r}}; \tag{13}$$

$$\frac{d\bar{p}}{d\bar{r}}=2\left[\frac{n_1}{(2+n_1)\left(\frac{n_1}{1+n_1}+\frac{n_2}{1+n_2}\right)^2}\right.$$
$$\left.+\frac{n_2}{(2+n_2)\left(\frac{n_1}{1+n_1}+\frac{n_2}{1+n_2}\right)}\right]\frac{\bar{q}^2}{\bar{r}^3}$$
$$+0.5\bar{r}\left[1-\frac{(n_1+3)n_1}{(1+n_1)(2+n_1)}+\frac{2n_1}{(1+n_1)(2+n_1)}y\right.$$
$$\left.+\left(\frac{n_1}{2+n_1}+\frac{n_2}{2+n_2}\right)y^2\right]+2^{\frac{3+n_1}{1+n_1}}A_1^{-\frac{2n_1}{1+n_1}}$$
$$\times\frac{(1+n_1)(1+n_2)}{n_1(1+n_1)+n_2(1+n_2)}(\bar{s}\bar{r}\mathrm{Re}_u)^{-\frac{2}{1+n_1}}\bar{q}$$
$$\times\left[(1-y)^2+\left(\frac{2\bar{q}}{\bar{r}^2\left(\frac{n_1}{1+n_1}+\frac{n_2}{1+n_2}\right)}\right)^2\right]^{\frac{n_1-1}{2(n_1+1)}}-2^{\frac{3+n_2}{1+n_2}}A_2^{-\frac{2n_2}{1+n_2}}$$
$$\times\frac{(1+n_1)(1+n_2)}{n_1(1+n_1)+n_2(1+n_2)}(\bar{s}\bar{r}\mathrm{Re}_u)^{-\frac{2}{1+n_2}}$$
$$\times\bar{q}\left[y^2+\left(\frac{2\bar{q}}{\bar{r}^2\left(\frac{n_1}{1+n_1}+\frac{n_2}{1+n_2}\right)}\right)^2\right]^{\frac{n_2-1}{2(n_2+1)}}. \tag{14}$$

The constants n_1 and A_1 describe the velocity profile in the region adjacent to the disk and the constants n_2 and A_2, that in the region adjacent to the wall.

The dependence of the constants n and A on Re is taken to be analogous to the corresponding dependence for a circular tube. In the range Re = 10^4-10^7, according to the data of [8], these dependences may be taken in the form

$$n=2\lg\mathrm{Re}-3;\quad A=1.8\lg\mathrm{Re}-0.26.$$

For the flow region adjacent to the disk, the Reynolds number is

$$\mathrm{Re}_{s_d}=\frac{w2s}{\nu}=2\mathrm{Re}_u\left[(1-y)^2+\frac{\bar{q}(n+1)^2}{\bar{r}^2n}\right]^{\frac{1}{2}}\bar{s}\bar{r}, \tag{15}$$

and for the flow region adjacent to the wall

$$\mathrm{Re}_{s_w}=\frac{U2s}{\nu}=2\mathrm{Re}_u\left[y^2+\left(\frac{\bar{q}(n+1)}{\bar{r}^2n}\right)^2\right]^{\frac{1}{2}}\bar{s}\bar{r}. \tag{16}$$

In the first approximation, the constant n may be taken equal to 7. As shown by calculation, this approximation is adequate for the required level of accuracy. As is evident from Eqs. (15) and (16), Re_{s_d} and Re_{s_w} depend on the flow swirl and the radius, which complicates Eqs. (13) and (14). In order to investigate the effect of change in Re along the radius, calculations were made for constant Re (determined at $\bar{r}$ = 1, i.e., at the radius at which the medium is supplied) and for Re changing at each integration step. These calculations showed that the error in determining the pressure drop over the radius when Re is taken to be constant over the radius amounted to ~10%.

In the case where the results of the calculation show that Re passes outside the range $10^4 \le \mathrm{Re} \le 10^7$ for a considerable part of the channel, the calculation must be repeated with new values of n and A, chosen in accordance with the flow conditions [8]. Calculations show that in the majority of cases Re does not pass outside the given range, although this is possible sometimes, on comparatively small parts of the channel.* The range of y for which Re < 10^4 is as follows: for flow close to the wall,

$$y^2 < \frac{\left(\frac{\nu}{2s}10^4\right)^2 - q^2\left(\frac{n-1}{2\pi rsn}\right)^2}{(\omega r)^2};$$

for flow close to the disk,

$$(y-1)^2 < \frac{\left(\frac{\nu}{2s}10^4\right)^2 - q^2\left(\frac{n+1}{2\pi rsn}\right)^2}{(\omega r)^2}.$$

A Nairi-2 digital computer was used to integrate Eqs. (13) and (14). Calculations were carried out for initial flow swirl 0.57 and 0.60 and various flow-rate coefficients $\bar{q}$ (in the range 0.08-0.007), corresponding to measurements made in an experimental investigation [6] of the flow between a rotating and a static disk. The results of the calculation are shown in Fig. 1, together with radial pressure distributions obtained experimentally.

As is evident from the above, for values of the flow-rate coefficient $\bar{q} \geq 0.02$, the calculated and experimental results are in satisfactory agreement. For $\bar{q} < 0.02$, the calculated radial pressure drop is larger than the experimental value. This is due to a lack of correspondence between the calculation scheme and the actual flow picture. At these values of $\bar{q}$, evidently, a significant role begins to be played by circulation of the medium in the gap, which is not taken into account in the chosen velocity profile.

In conclusion, note that the velocity profile of the flow used in [5] makes allowance for circulation of the medium in the gap. However, the use of this profile leads to the appearance in the calculation, under certain conditions, of a negative value of the flow swirl at the gap midpoint, which is inconsistent with the physical picture of the problem. For comparison, Fig. 2 shows results calculated by the method outlined above and by that of [5] for the flow characteristics with flow swirl 0.6, relative gap 0.0725, and flow-rate coefficient $\bar{q}$ = 0.01 and 0.04. For $\bar{q} = 0.04$ the results coincide, whereas for $\bar{q} = 0.01$ the flow swirl and pressure drop given by [5] are considerably lower.

The conclusions are as follows.

1. A method has been developed for the calculation of the flow swirl and radial pressure drop in the gap between a rotating and a static disk in the presence of radial flow. There is good agreement of the calculated and experimental results for $\bar{q} \geq 0.02$.

2. For $\bar{q} < 0.02$, the discrepancy between the experimental and calculated data increases, evidently as a result of circulation of the medium in the gap between the disks. To solve the problem in this range of flow rates, additional investigations are required.

NOTATION

r, φ, z, radial, azimuthal, and axial coordinates; s, gap width between disks; δ, δ', thicknesses of boundary layers at static and rotating disks; ρ, density; p, pressure; q, volume flow rate of medium in radial direction; ν, kinematic viscosity; u, w, absolute and relative velocities; c_r, c_φ, components of absolute velocity; ω, angular velocity; U, flow velocity at gap midpoint; $c^* = \sqrt{\tau_0/\rho}$, dynamic flow velocity; τ_{zr}, $\tau_{z\varphi}$, shear-stress components; $\bar{s} = s/r_2$; relative gap width; $\bar{r} = r/r_2$, relative radius; $\bar{q} = q/2\pi r_2^2 \omega s_2$, relative flow rate; $\bar{p} = p/\rho\omega^2 r_2^2$, relative pressure; $y = c_\varphi/\omega r$, flow swirl; $Re_u = \omega r_2^2/\nu$, Reynolds number; A_1, A_2, n_1, n_2, coefficients. Indices: d, disk; w, wall.

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*In the case of flow from the periphery to the center of the disk for $\bar{q} = 0.01$ and $\bar{s} = 0.01$ (see Fig. 2), in the flow region close to the disk, the chosen resistance law is not satisfied only on the section $\bar{r} = \pm 0.01$, which has no effect on the accuracy of the calculation.

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BEHAVIOR OF FILM OF VISCOPLASTIC LIQUID IN THE PRESENCE OF SLIP AT THE WALL

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The shaking loose of a viscoplastic film of limiting thickness from a plane surface is considered for the case when there is effective slip at the wall.

A film of viscoplastic liquid is characterized by a limiting value of the thickness, at which no flow is observed under the action of gravity. This limiting thickness is found from the balance of frictional and gravitational forces

$$h = \tau_0/\rho g,$$

where τ_0 is the yield point; ρ is the density; and g is the acceleration of gravity.

In a number of technological processes, it is necessary to prevent the formation of a liquid film at a wall. The present paper considers a dynamic approach to this problem, by vibration of the wall.

Suppose that the wall and the adhering film are moving uniformly downward with velocity U and, at the initial moment t = 0, stop instantaneously. Close to the wall, the stress exceeds the yield point τ_0, which leads to the formation of a region of viscoplastic flow. In the second region, where the stress is less than τ_0, the liquid moves in a quasisolid manner. In the immediate vicinity of the wall, the moving disperse system, or polymer solution, may be separated into a thin layer of solvent, with respect to which all the remaining mass slips, as in a lubricant. It is possible to neglect the thickness of the region at the wall in comparison with the film thickness and to assume that at the surface of the plate the adhesion hypothesis does not hold, i. e., there is effective slip at the wall, $u(0, t) \neq 0$. (The case $u(0, t) = 0$ was considered in [1].)

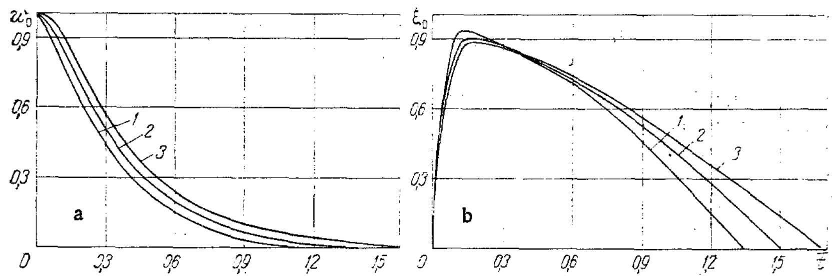

Fig. 1. Velocity of quasisolid core of film flow (a) and boundary of quasisolid region (b) for S = 0.25: 1) P = 0; 2) 0.05; 3) 0.1.

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